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Parameter estimation in nonlinear time-varying systems through Takagi-Sugeno fuzzy models and wavelets

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Abstract

In this paper, a parameter estimation problem for a Takagi-Sugeno fuzzy dynamic system is formulated under the assumption that the premises in the membership functions are known. A linear expression in consequent parameters is obtained under this assumption. If the system is time-varying, the parameters can be determined by recursive estimation techniques. As an alternate approach, the use of multi-resolution wavelets is proposed. Furthermore, a parameter estimation toolbox for fuzzy dynamic models is developed which is then applied to a simple example and to the Mackey-Glass chaotic time series.

Keywords: Wavelets, Takagi-Sugeno Models, Fuzzy Models, Time-Varying Systems, Parameter Estimation.

1 Introduction

There are two known methodologies for model based fault detection and diagnosis. The first one consists in the comparison of normal behavior patterns with respect to fault situations [Basseville, 1993], [Evsukoff, 2001]. The second consists in the analysis of signals generated (residues) in the different scenarios. For these purposes observers or Kalman filters are generally employed [Patton, 1989].

The design of these methodologies requires the availability of a dynamic model of the process, whose parameters are estimated from the input and output data [Ljung, 1999]. If the process or the system is non-linear, then polynomial (Volterra, Hammerstein or Wiener, [Westwick, 1995]), neuronal [Mastorocostas, 2002], or fuzzy models [Johansen, 2000], [Schiavo, 2000] can be used.

This paper considers the case of non-linear time-varying systems. In this situation, the literature presents two approaches. The first of them employs recursive estimation, such as recursive least squares (RLS) methods or Kalman filters, [Tsypkin, 1992], [Niedzwiecki, 1994], [Chowdhury, 2000]. The second is based on the representation of time-varying parameters through time-invariant basis series, which converts the TV problem into an invariant one [Grenier, 1983], [Doroslovacki, 1998], [Eom, 1999].

In order to deal with system non-linearities, Takagi-Sugeno dynamic fuzzy models [Takagi, 1985], [Johansen, 2000], [Ho, 2001] are proposed. To take into consideration the time-varying characteristics of the system, it is assumed that the consequent parameters change with time. Wavelets [Tsatsanis, 1993] are used as an alternative to recursive estimation.

2 Takagi-Sugeno type dynamic models

Given a non-linear dynamic system with input *u(t)* and output *y(t)*, a Takagi-Sugeno model is described by the following *R* rules [Jamshidi, 1993]:

Rule r, r=1,...,R:

`if`
$y(k-1)$ `is` $\pi_{1,r}$ `and ... and` $y(k-n_a)$ `is` $\pi_{n_a,r}$
`and` (1)
$u(k-1-d)$ `is` $\pi_{n_a+1,r}$ `and ...` $u(k-n_b-d)$ `is` $\pi_{n,r}$
`then`

$$y(k) = y_r(k)$$

In the last expression, the term $y_r(k)$ refers to the output of an ARX (Auto-Regressive with eXogenous inputs) sub-model, defined by:

$$y_r(k) = \sum_{i=1}^{n_a} a_{i,r}\, y(k-i) + \sum_{j=1}^{n_b} b_{j,r}\, u(k-j-d) \qquad (2)$$

where d is the input delay, and $n=n_a+n_b$ is the total number of parameters in each sub-model.

The term $\pi_{i,r}$ denotes the fuzzy set of the r^{th} rule and i^{th} term in the sub-model. For each fuzzy set, it corresponds a membership function $\mu_{\pi_{i,r}}$. The rule activation degree β_r of the r^{th} rule at instant k is given by:

$$\beta_r(k) = \prod_{i=1}^{n_a} \mu_{\pi_{i,r}}(y(k-i)) \prod_{j=1}^{n_b} \mu_{\pi_{na+j,r}}(u(k-j-d)) \qquad (3)$$

where $\prod$ represents the fuzzy `OR` operator. Using fuzzy inference, the following expression is obtained [Jamshidi, 1993]:

$$y(k) = \sum_{r=1}^{R} \omega_r(k) \left(\sum_{i=1}^{n_a} a_{i,r} y(k-i) + \sum_{j=1}^{n_b} b_{j,r} u(k-j-d) \right)$$
$$y(k) = Z(k)f \tag{4}$$

where:

$$\omega_r(k) = \frac{\beta_r(k)}{\sum_{r'=1}^{R} \beta_{r'}(k)}$$
$$Z(k) = \omega_1(k)y(k-1)\Lambda\ \omega_R(k)y(k-n_a)\Lambda\ \omega_1(k)u(k-1-d)\Lambda\ \omega_R(k)u(k-n_b-d) \tag{5}$$
$$f = \left[a_{1,1}\ \Lambda\ a_{n_a,R}\ \Lambda\ b_{0,1}\ \Lambda\ b_{n_b-1,R}\right]^T$$

If the membership functions $\mu_{\pi_{i,r}}$ are considered known and if a sufficient number of present and past input *u(k)* and output *y(k)* values are available, then it is possible to apply (4) and determine the consequent parameters vector *P* using a parameter estimation method [Takagi, 1985].

3 Identification of time-varying fuzzy dynamic models

In fuzzy time-varying dynamic models, the consequents coefficients can change with time. In this case, expression (4) is useful for the estimation of those parameters, through recursive techniques [Ljung, 1999].

This paper proposes the use of wavelets for the time-varying parameters estimation [Tsatsanis, 1993], [Galvao, 2002]. The wavelet analysis allows representing signals in terms of coefficients that express both variability in time and variation speed [Ho, 2001]. Thus, the analysis of changes in the signals can be done with a reduced number of coefficients of a wavelet decomposition. Another advantage of the wavelet analysis is that the nature of the approximation of signals through the wavelet basis is specially suited for characterizing the abrupt changes and faults in dynamic systems [Lada, 2000].

In the following section, the application of wavelets in parameter estimation is briefly described.

4 Time-varying parameter estimation through wavelets

4.1 Background in wavelets

Wavelet analysis emerges as a natural extension of Fourier analysis for the approximation of signals in $L^2(\Re)$. In this case, the "mother function" for the space basis is not a sinusoid but a *wavelet* [Chui, 1992a], [Schumaker, 1994].

Thus, it is possible to approximate signals through the shifts and scalings of a mother wavelet function ψ of the form $\psi_{j,i}(t) = 2^{j/2}\psi(2^j t - i), j,i \in Z$, which allows to express any function *f(t)* as a *wavelet series*:

$$f(t) = \sum_{j,i=-\infty}^{\infty} c_{j,i} \psi_{j,i}(t) \tag{6}$$

The selection of the mother function ψ is not trivial. Frequently, stability conditions are imposed over spectrum of functions $\psi_{j,i}(t)$, convergence of the series (6), and orthogonal relationships among the various spaces generated by $\psi_{j,i}(t)$ [Chui, 1992b], [Schumaker, 1994].

4.2 Matrix representation of the multi-resolution analysis

The family function $\psi_{j,i}(t)$ described induces multiple levels of decomposition in the spaces $W_j = \langle \psi(2^j t - i) \rangle$. A function ϕ is called a *scaling function* if it is possible to obtain the decompositions over the spaces $W_j, j < j_0$ as a unique decomposition over $V_j = \langle \phi(2^j t - i) \rangle$. Thus, based on the functions ψ and ϕ, it is possible to decompose signals as [Chui, 1992a]:

$$f^{j_1}(t) = f^{j_0}(t) + \sum_{j=j_0}^{j_1-1} g^j(t) \tag{7}$$

where $f^j(t)$ is the projection of the function $f(t)$ over V_j, and $g^j(t)$ is the projection of $f(t)$ over W_j. The integers j_0 and j_1 define upper and lower levels for the decomposition.

In the following, time-discretized signals are considered, which are represented as column vectors. Since $f^j(k)$ and $g^j(k)$ are projections over V_j and W_j respectively, we can write:

$$f^J(k) = \sum_i c_i^J \phi(2^J k - i) = \phi^J(k) c^J \tag{8}$$

$$g^J(k) = \sum_i d_i^J \psi(2^J k - i) = \psi^J(k) d^J \tag{9}$$

In the last expressions, the terms c^J and d^J are wavelet coefficients vectors, commonly called approximation and detail coefficients, respectively. Additionally, the terms ϕ^J and ψ^J are wavelet matrices. For simplicity, the details of its composition are omitted, but it is possible to show that they has the form:

$$\phi^J = \prod_{j=1}^{J} A^j, \quad \psi^J = \prod_{j=1}^{J-1} A^j B^J \tag{10}$$

where A^j and B^j matrices contain scalings and shiftings of the functions ψ and ϕ as columns. Then:

$$f^{j_1}(k) = \phi^{j_0}(k) c^{j_0} + \sum_{j=j_0}^{j_1} \psi^j(k) d^j = \Phi(k) C \tag{11}$$

where:

$$\Phi(k) = \left[\phi^{j_0}(k) \psi^{j_0}(k) \Lambda\ \psi^{j_1}(k)\right]$$
$$C = \left[c^{j_0 T} d^{j_0 T} \Lambda\ d^{j_1 T}\right]^T \tag{12}$$

4.3 Parameters in a fuzzy Takagi-Sugeno type system

If we assume that $f(t)$ is a generic parameter of a time-varying system, then (11) is an approximation of $f(t)$ by $f^{j1}(t)$, which corresponds to its multi-resolution decomposition at j_1th level.

Particularly, in a time-varying fuzzy model, each consequent parameter can be expressed as a wavelet decomposition. Applying the decomposition (11) over the fuzzy model (4), we obtain:

$$y(k) = Z(k)\Phi(k)C \tag{13}$$

Therefore, considering a time series in *k*, the problem of determining the parameter vector *f* can be reduced to a regression problem in C, given that *f* and *C* are related in an approximate manner by (11).

4.4 Selection of the decomposition structure for parameter estimation

From equation (13), it is possible to show that the matrix $Z\Phi$ is a N by $R \cdot n \cdot n_p$ block, where n_p is the number of terms in each wavelet expansion. Since n_p can be expressed as a fraction $\alpha \in (0,1)$ of the length of data vectors N, the regression problem (13) generates infinite solutions for *C*, which is numerically expressed as an ill-conditioned pseudoinverse matrix.

In this case, the conditioning norm for $\Theta = (Z\Phi)^T Z\Phi$ [MathWorks, 2002]:

$$cond(\Theta) = \|\Theta\| \|\Theta^{-1}\| \tag{14}$$

takes considerably big values, instead of values near 1. This can be interpreted as numerical deviations in the pseudo-inverse Θ^{-1} respect to a rectangular inverse for $Z\Phi$. In (14), $||\cdot||$ is the norm obtained from taking the highest value of the diagonal matrix from the singular value decomposition (SVD) [MathWorks, 2002].

This shows that the use of the pseudo-inverse matrix in (13) is not an appropriated method for the estimation of *C*.

In this way, the selection of coefficients in the wavelet decomposition is proposed as a solution for the conditioning problem [Tsatsanis, 1993]. The initial search space consists of the terms associated to the projections in j_0-j_1+1 wavelet spaces. Then, to reduce the search space, Multiple Hypothesis Tests are performed over different decomposition structures. In [Lada, 2000] various alternatives to perform such tests are mentioned.

The Hypothesis Test formulated between two models is:

$$\begin{aligned} H_0 &: \quad C = C_1 \\ H_1 &: \quad C = C_2 \end{aligned} \tag{15}$$

where C_i is a vector of coefficients of length d_i, which defines the wavelet decomposition if the hypothesis H_i is true.

For two decompositions with equal number of coefficients, the hypothesis are evaluated using the mean square error $E_i = \sum_k (y(k) - \hat{y}_i(k))^2$ corresponding to the obtained parameters, with $\hat{y}_i(k)$ being the estimation of $y(k)$ using the decomposition C_i.

When two decompositions have different number of coefficients, the test is decided through:

$$t(E_2, E_1, d_2, d_1) = \frac{N - d_2}{d_2 - d_1} \frac{E_1 - E_2}{E_2} \tag{16}$$

According what it was stated, Figure 1 shows the algorithm used to select between two models. In this work, different schemes for selection of coefficients are tested. The following are among them:

- Successive addition of coefficients.
- Successive addition of coefficients with subtraction cycles.
- Monte Carlo search.
- Fixed size models.

The first three schemes are initialized with a model without coefficients; this is done in order to further increment the number of parameters in the structure. The fixed size technique does not require search.

Finally, the decomposition structure obtained is fed into a multiple regression algorithm. The regression problem is solved through to the following equations [MathWorks, 2002]:

$$\begin{aligned} \delta y &= y - AR^{-1}R^{T^{-1}}A^T y \\ \delta C &= R^{-1}R^{T^{-1}}A^T \delta y \\ C &= R^{-1}R^{T^{-1}}A^T y + \delta C \end{aligned} \qquad (17)$$

where R is the orthogonal matrix obtained by the QR decomposition of $Z\Phi$.

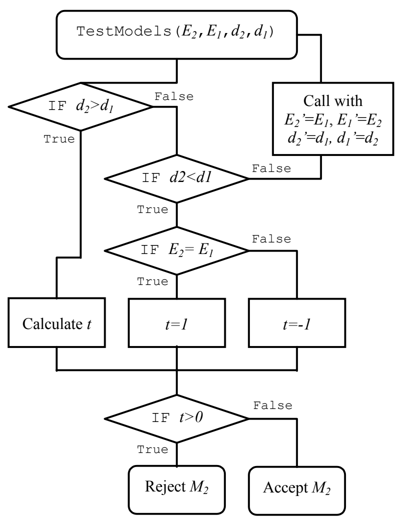

Fig. 1. Selection between two models

5 Toolbox for parameter estimation for time varying systems

A non-linear time-varying parameter estimation MATLAB toolbox was developed based on the previous concepts. Its basic data structures are presented in Appendix A.1, while Appendix A.2 presents its main functionalities.

The following sections describe two applications of the toolbox oriented to non-linear time-varying systems.

5.1 Example 1: Fuzzy dynamic system with time-varying parameters

This first example is included for testing purposes. Let's consider the following two rule TS fuzzy dynamic system:

Rule r, r=1,2:
`if`
$y(k-1)$ `is` $\pi_{1,r}$
`then`
$y(k) = a_{1,r} y(k-1) + b_{1,r} u(k-1) + e_r(k)$

The parameters $a_{1,r}$ and $b_{1,r}$ vary with time; disturbances $e_r(k)$ are non-correlated white noise signals with a standard deviation $\sigma_{e_r} = 0.01$. The fuzzy sets $\pi_{1,1}$ and $\pi_{1,2}$ are characterized by two symmetric triangular membership functions on the interval [0,1].

Figure 2 shows the system's input and output signals obtained for an initial condition $y(0)=1$. The input $u(k)$ is a PRBS (Pseudo Random Binary Sequence) signal with $u_{min}=0.05$, $u_{max}=0.45$, registry length 2^{11}, and clock time $\tau = 1$.

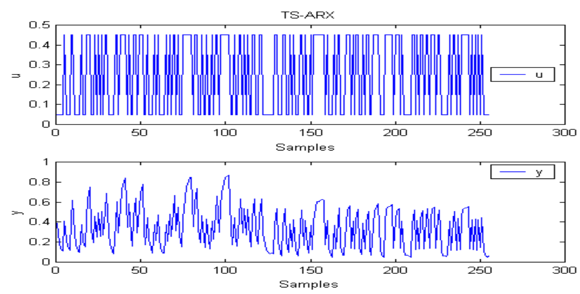

Fig. 2. Input and output signals

Figures 3 and 4 show the time evolution of the process and the estimated parameters. For such, the function `FuzzWavEst` present in the toolbox was employed, selecting in this case the wavelet family `db1` (First order Daubechies). Additionally, the limits of the decomposition were set at `Jmin=7` and `Jmax=10`. For structure selection, the fixed size models method has been used.

Table 1 presents the absolute and relative RMS errors of each estimated parameter, with the input and output signals shown in Figure 2. The mean relative RMS error oscillates between 0.0171 and 0.086. Such indicates a good determination of the unknown parameters if is considerated that the estimation has been performed by minimizing the estimation RMS error of the output $y(k)$.

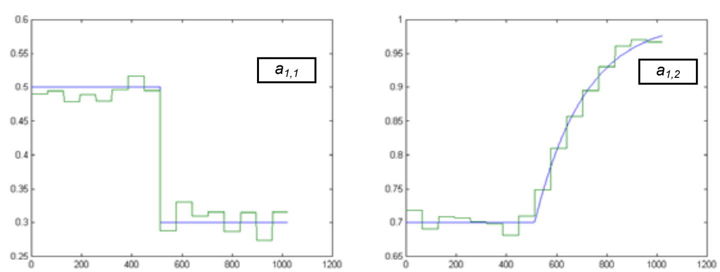

Fig. 3. Real (solid) and estimated (dashed) parameters $a_{1,1}$ and $a_{1,2}$

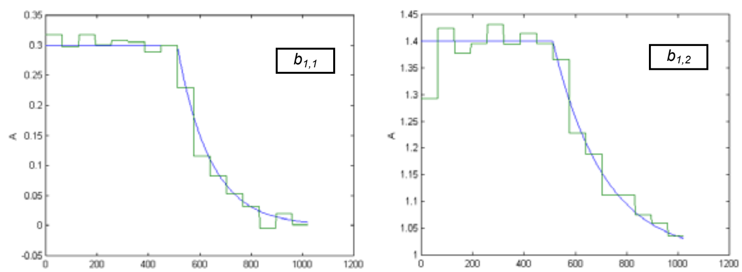

Fig. 4. Real (solid) and estimated (dashed) parameters $b_{1,1}$ and $b_{1,2}$

Table 1: Estimation errors in Example 1

Parameter	*Mean*	*RMS*	RMS_{REL}
$a_{1,1}$	0.397	0.0161	0.0405
$a_{1,2}$	0.737	0.0126	0.0171
$b_{1,1}$	0.186	0.0160	0.0860
$b_{1,2}$	1.350	0.0331	0.0245

5.2 Example 2: Mackey-Glass chaotic series

The Mackey-Glass chaotic series is defined by the following differential equation, [Yamakawa, 1994], [Schiavo, 2000]:

$$\dot{x}(t) = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.2x(t) \tag{18}$$

with $\tau = 17$ and $x(0)=1.2$. For the data generation, the $x(t)$ variation between $t=0$ and $t=1000$ with sample time $\Delta t = 1$ is considered (see Figure 5).

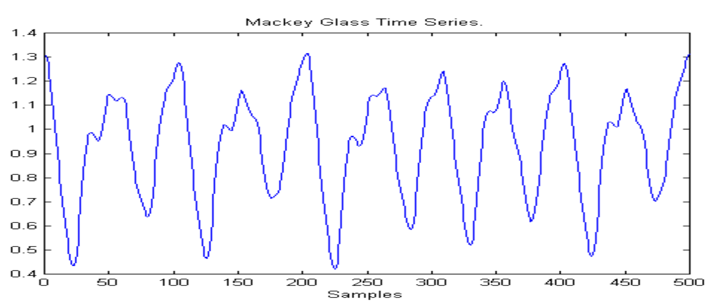

Fig. 5. Evolution of output of Mackey-Glass chaotic series

For estimation purposes, it is necessary to transform the original autonomous system into a discrete-time system with input u(k). In that sense, the following variables are considered as inputs:

$$u(k) = [x(k-18)\ x(k-12)\ x(k-6)\ x(k)]$$

and the output is given by:

$$y(k) = [x(k+6)]$$

The generation of data for estimation is made considering the first 500 samples, starting from k=118.

In this case, the fuzzy sets for the model have been determined using fuzzy clustering (Fuzzy C-Means) through the `MakeFStr` Toolbox utility. In this manner, the structure of the obtained fuzzy model for 3 clusters is the following:

Rule r, r=1,...,3:

if
$x(k-18)$ is $\pi_{1,r}$ and $x(k-12)$ is $\pi_{2,r}$ and
$x(k-6)$ is $\pi_{3,r}$ and $x(k)$ is $\pi_{4,r}$
then
$y(k) = a_{1,r}x(k-18) + a_{2,r}x(k-12) + a_{3,r}x(k-6) + a_{4,r}x(k)$

Figure 6 shows the evolution in time of the estimated parameters for the first fuzzy rule using `FuzzWavEst`. The figure also shows the performance of the Toolbox for the `db2` wavelet family (Second order Daubechies) with decomposition limit levels `Jmin=7` and `Jmax=9`.

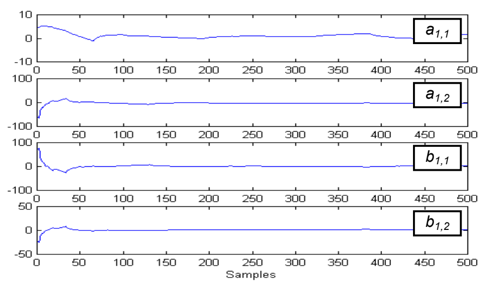

Fig. 6. Estimated parameters for the first fuzzy rule

Figure 7 shows the estimation error variation. For the considerated data, the RMS error is 0.0475, equivalent to a mean relative RMS error of 0.051. In this case the parameter estimation also performs a good fit, even-though the structure of the plant is unknown.

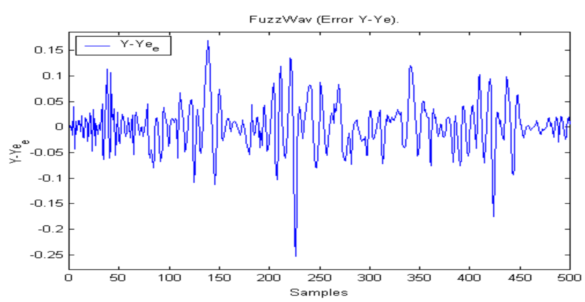

Fig. 7. Estimation error

For validating the parameter estimation process, the initial condition of the autonomous system has been modified from *x(0)*=1.2 to *x(0)*=0.9. The results are shown in Figure 8 for the first 500 data samples from *k*=118. It is shown that the simulation with the new data based on the varying parameters generates a prediction RMS error of 0.190, which is equivalent to a relative RMS error of 0.204.

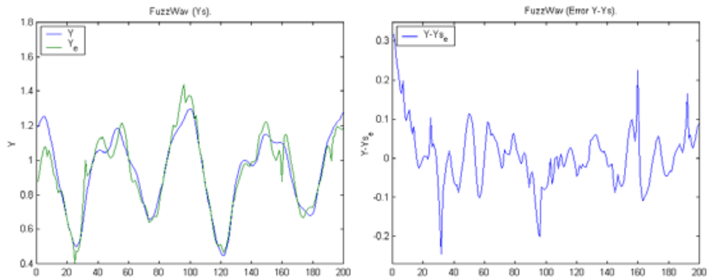

Fig. 8. Real (solid) and predicted (dashed) output, and prediction error

6 Conclusions

This work introduces a parameter estimation method applicable to non-linear time-varying dynamic systems. A toolbox was developed for such purpose that utilizes non-linear fuzzy models and parameter estimation based on wavelets.

In the first example was shown that the time evolution of the estimated parameters corresponds to the parameters of the real TS model, which are supposedly unknown. In the second example, reduced estimation errors were obtained. The data validation indicates that the modelling through time-varying parameters represents reasonably the process dynamics independently of the initial condition introduced.

In the special case of the Daubechies wavelet family, due to the discontinue nature of the basis functions, it is possible to obtain adequate trajectories for estimated parameters in presence of abrupt and smooth changes.

A topic under current research by the authors is the application of the method to on-line prediction using recursive parameter estimation. Also, applications to fault detection and diagnosis in simulated and real processes are under investigation.

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Appendix: Toolbox Structure

A.1 Toolbox basic data structures

The following paragraphs present the data structures used in this toolbox.

`N`: Number of samples considered for the data analysis.

`R`: Number of rules of the fuzzy model considered, and also number of clusters in a model built on fuzzy sets.

`n`: Number of variables on the Input Data.

`TH`: System input data matrix.

`Y`: System output data column vector.

`FStr`: Data structure for fuzzy sets representation in a Takagi-Sugeno model.

`WH`: Fuzzy activation degrees matrix for each sample and fuzzy rule.

`Str`: Wavelet family.

`Jmin`, `Jmax`: Minimum and maximum levels for a wavelet decomposition.

`Typestr`: Coefficient class. `typestr='a'`: approximation coefficients. `typestr='d'`: detail coefficients.

`C`: Conditioning number for the main problem matrix X, equal to $\|X\|\|X^{-1}\|$.

`lopt`, `laux`, `l`: Vectors for obtained, auxiliary and initial decomposition structure, respectively.

`Ye`: Estimated output data.

`Ae`: Estimated parameters for the chosen Takagi-Sugeno structure.

A.2 Basic functions of the Toolbox

The following lines present the prototypes of the principal developed functions.

`[Ye,Ae,c,lopt,l]=FuzzWavEst(TH,WH,Y,str,Jmin,Jmax):`
Parameter estimation for Takagi-Sugeno models for nonlinear time-varying systems, using wavelet multi-resolution decomposition.

`[Ye,Ae,c]=ArxWavEst(TH,Y,str,Jmin,Jmax):`
Parameter estimation for ARX models for nonlinear time-varying systems, using one shot least squares techniques.

`[TH,Y,A]=MakeExample<i>(N):`
Data generation routines of nonlinear time-varying systems. `<i>=1` corresponds to an ARX system, `<i>=2` to a TS-ARX system and `<i>=3` to the Mackey-Glass autonomous system.

`[FStr,WH]=MakeFStr(TH,R)`:

Generation of the premises structure for a Takagi-Sugeno model based on input data and desired number of rules. Fuzzy Clustering (C-Means) is applied using the `fcm` utility, exported from the MATLAB© Fuzzy Logic Toolbox©.

`Pdemo<i>`:

Demos for `FuzzWavEst` tool.

`[h,t]=TestModels(MSE2,MSE1,d2,d1,N)`:

Perform an hypothesis test between two wavelet decomposition models based on its mean square errors `MSE1` and `MSE2` and its number of parameters `d1` and `d2`. `h=0` implies that the model 2 is rejected, and `h=1` indicates that such model is accepted.