

# A mathematical model for the optimization of forest planning decisions: evaluation of the trade-offs between timber production and carbon sequestration

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## Abstract.

A forest planning decision support system assists managers by providing a sequence of activities that each forest land area will receive through the whole planning horizon, as well as the schedule concerning all these activities are required to be performed. Each activity is a composition of silviculture treatments, and different combinations of activities are possible, which in turn depends on the type of trees. Also, the yield and operational costs are determined by the tree species, age, previous received treatments, among other features.

In order to tackle the complexity of the optimal forest planning problem, a general mathematical framework based on a Generalized Disjunctive Programming (GDP) approach is proposed. The approach defines (i) the optimal combination of silvicultural treatments (a forest management plan), (ii) the proportion of land area to be harvested, and (iii) the volume of timber products produced at each harvest node. The maximization of the net present value and the minimization of carbon sequestration lost are used as a multi-objective function.

The  $\epsilon$ -constraint methods are used to evaluate the trade-offs between aforementioned objectives functions, which can be used with the aim of supporting forest planning management decisions. Data from a forestry industry in the northern province of Misiones (Argentina) are used to test the applicability of the proposed model.

**Keywords:** Forestry planning, Generalized Disjunctive Programming, Carbon sequestration.

## 1 Introduction

The present work addresses the long-term forest planning problem (FPP), where several forestry operations, such as pre-commercial thinning and clear-cutting among others, needs to be scheduled in order to reach predefined goals.

There exist many contributions that presents different types of forest management decision supporting tools [1]. Even though some recent works integrate two sets of decisions [2], there is not any contribution tackling simultaneously, as a multi-objective function, the maximization of the net present value and the minimization of the carbon sequestration lost.

Most works addressing the FPP uses linear and mixed 0-1 approaches to formulate the problem. In this proposal, a Generalized Disjunctive Programming (GDP) is employed as modelling technique to cope with the complexity of the problem.

## 2 Problem statement

The FPP addressed in this work concerns with the development of a forest plan for a predefined area comprised by several stands. Through the whole planning horizon, each stand is managed using forest management prescriptions. Each prescription is an ordered sequence of forest operation that must be performed on a specific stand age (a set of pre-commercial thinning, and clear-cutting). There are many different products which are produced by each forest operation.

At the beginning of the planning horizon, the characteristics of the stands are heterogeneous, therefore, they are grouped into homogeneous strata, i.e., their elements share common characteristics such as species, age, and previous received treatments.

The forest planning decision problem addressed can be described as follows:

*Given:* a set of forest stands, a set of forest management prescriptions, a set of forest operations, a set of tree species, the unit costs of production of each forestry operation, the yield of timber production, the age and previous received forest operations per each stand.

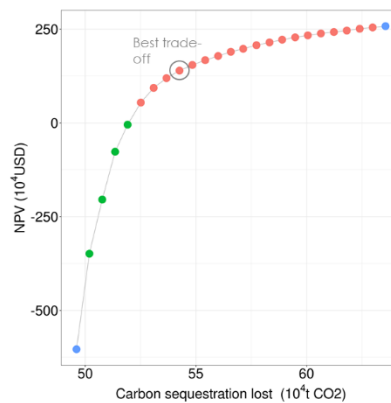
*Determine:* the optimal scheduling of forest management alternatives (combination of forestry operations) for each stand, and their timber assortment production in order to *Maximize the Net Present Value (NPV)* and *Minimize Carbon Sequestration Lost (CSL)* that are caused by the reduction of forest cover as a result of the execution of forestry operations.

The proposal assumes that: (i) forest prescriptions are known beforehand, (ii) clear-cutting is the last possible operation, (iii) a prescription ends when its last prescribed operation is completed, (iv) for stablished stands at the beginning of the planning horizon, a set of potential forest management prescriptions that can be first assigned is differentiated from another set of prescriptions that can be scheduled later, and (v) the stakeholders' demand must be met, and surplus can be sent to third parties. The formulation is based on [3] and is not present in this manuscript because of lack of space. In the previous work [3], the minimization of the absolute deviations of timber assortment production between consecutive periods was considered as the second objective function. Here, *CSL* is considered, which is the main difference between the two works.

### 3 Case study and results

In order to demonstrate the applicability of the model, a case study representative of the Argentine forest sector was developed. Forest management prescriptions are obtained from the previous work [3], in which the SisPinus forest growth simulator was used, using real data from a forestry company located in the north of the province of Misiones. As a result, 30 different forest management prescriptions were obtained, considering a forest asset of 26 stands planted with *Pinus Taeda* that cover an area of 483.88 ha. The generated prescriptions include three pre-commercial thinning to be carried out between 7 and 13 years of stand age, and clear-cutting to be carried out between 16 and 20 years of stand age. A planning horizon of 30 years was assumed.

To determine efficient Pareto optimal solutions, the economic (*NPV*) and environmental (*CSL*) performance of each of the 25 Pareto optimal solutions are measured. The Pareto set is plotted in **Fig. 1** to facilitate graphical visualization and analysis. One of the main observations made is the clear trade-offs between the considered objectives. The best values for *NPV* and *CSL* are obtained optimizing each metric alone and disregarding the other. These executions give the two extreme values shown in blue. For the rest of the solutions, the *CSL* is constrained and the *NPV* is optimized. The four green points indicate solutions with negative *NPV*, while positive *NPV* solutions are presented in red points. It is observed that by moving to the right from the environmentally optimal solution on the curve, significant increases in *NPV* can be first achieved up to a point where only small economic improvements are obtained. When analyzing the relative improvements in each of the red points, i.e., the economically profitable solutions, the red point with a grey circle is considered to be the best compromise solution.



**Fig. 1.** Views of the Pareto-optimal solutions.

**Fig. 2.** shows the timber production for the best compromise solution. It is observed that at the beginning of the planning horizon there is a predominance of first and second thinnings. Meanwhile, from year 10 of the planning horizon, the clearing process takes place more intensively. This behavior is consistent with the age distribution of the forest inventory that was considered in the case study. For example, in the first planning period, given the characteristics of the stands (i.e., age and forest operations performed

prior to the start of the planning horizon), all allowable prescriptions to be assigned to the stands begin with the second thinning operation. That is, the stands that were thinned for the second time in the first planning period had previously been thinned for the first time before the start of the planning horizon.

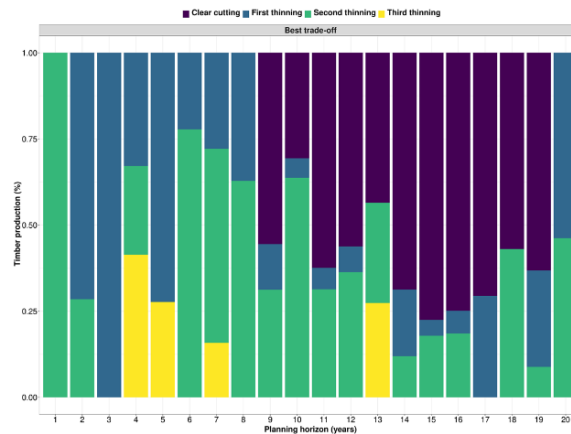


Fig. 2. Production per forest operation for the best compromise solution.

## 4 Conclusions

The resulting multi-objective optimization model was applied to a case study in a forest-dependent area located in the interior region of Misiones, Argentina, where the production of four different wood assortments are considered taking into account their diameter class. The  $\epsilon$ -constraint method was used to generate a set of Pareto optimal solutions considering the two objectives. Trade-offs between economic performance and its potential to generate environmental benefits were revealed by analyzing the Pareto-optimal solutions.

Future research could also include dealing with uncertainty in the model parameters, as well as integrating the forest planning problem with the forest biorefinery supply chain design problem.

## References

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