Estimating of time-dependent travel times vía Mixed Integer Programming

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Abstract. Routing and distribution problems have been widely studied within the Operations Research (OR) community. When restricting to distribution problems in large cities, the congestion of the road network becomes a key aspect with a significant practical impact. These problems are known as Time-Dependent VRPs (TDVRPs), as they naturally capture the effect of congestion by assuming that the travel time between any two customers varies depending on the departure time. The TDVRP literature has widely accepted to model the time-dependent travel time model between two customers as continuous piecewise linear (PWL) function that satisfies the first-in first-out (FIFO) condition. In this paper, we investigate the problem of estimating these continuous PWL travel time functions from real data travel time data. We benchmark two recently proposed Mixed Integer Programming based models for estimating general PWL functions and a well-known heuristic proposed within the context of travel-time estimations. In addition, we also contribute with a new dataset of instances created using real-world data as input.

Keywords: travel time estimation · piecewise linear functions · mixed integer programming

The recent growth of direct-to-consumer deliveries has stressed the importance of last-mile logistics, becoming one of the critical factors in city planning. According to different studies, one of the key factors lies in the last-mile deliveries, reaching in some cases nearly 50% of the overall parcel delivery cost. From a practical perspective, these technologies have been increasingly adopted and becoming part of the core business of large retailing, logistics and many other companies during the last years. However, the potential impact of improving logistics decisions at a big scale, both from an economical and environmental aspect, is still underestimated compared to other technologies. In the context of Smart Cities, exploiting real-time data collected through mobile and other devices could allow to move a step forward and enable advanced algorithms to improve routing decisions massively.

Routing and distribution problems have been widely studied within the Operations Research (OR) community. The classical vehicle routing problem with time windows (VRPTW) addresses a key operational constraint regarding the feasibility of the route, in particular related to the time when a customer must be visited. Briefly, the VRPTW involves the management of a fleet of vehicles which must satisfy a set of request under different operational constraints, such as vehicle capacities, time windows and several other resource constraints. The objective is to find a distribution plan of minimum operational cost. Providing efficient solution tools for real world distribution problems is a challenging problem, given that the VRPTW and many of its variants are $\mathcal{NP}$-hard optimization problems (see, e.g. [5]).

When restricting to distribution problems in large cities, the congestion of the road network becomes a key aspect with a significant practical impact. Accounting effectively for the effects of congestion at the planning level results in more accurate distributions plans, while neglecting its impact can lead to low quality routes, violating important operational constraints. Recently, more complex travel time functions have been considered to capture the impact of congestion by assuming that the travel time between any two vertices does not remain constant along the planning and may suffer variations depending on the moment the trip begins. The *time-dependent* travel time model proposed by [2] has been widely adopted in the literature, where the travel time between two customers is modelled as continuous piecewise linear (PWL) function that satisfies the first-in first-out (FIFO) condition. These problems are known as *Time-Dependent* VRPs (TDVRPs), as they naturally capture the effect of congestion by assuming that the travel time between any two customers varies depending on the departure time.

Most of the OR literature tackling TDVRPs is focused on the routing problem, assuming that the time-dependent travel information is part of the input. Indeed, the methods are in general evaluated over synthetic benchmark instances, which are used to establish fair methodological and algorithmic comparisons. From a more applied standpoint, translating real data into accurate estimations of the variable travel times represents a critical step towards an effective practical solution. In order to provide a complete framework suitable to apply these techniques, one of the earliest steps in the pipeline involves the computation of these continuous PWL travel time functions from real data between every pair of requests.

The problem can be stated as follows. Consider two locations (i, j) and a discrete series $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$, $x_0 < x_1 < \dots < x_n$, where y_t indicates the travel time of from i to j if departing from i at time x_t. Typically, the series may consist of travel times estimations with a granularity of 5 minutes for the entire planning horizon. Based on this input, the aim is to infer a continuous PWL function $\tau_{ij}(t)$, satisfying the FIFO condition, representing the possible variable travel times between i and j along the planning horizon. Intuitively, $\tau_{ij}(t)$ captures the fact that travel times may differ along the planning horizon. In addition, as these travel times are estimates and in general used within tactical planning, it is not necessary for $\tau_{ij}(t)$ to interpolate these values.

To the best of our knowledge the only approach tackling this problem from a logistics perspective is the heuristic approach proposed in [1]. Briefly, they propose a heuristic approach that (i) first applies a *smoothing* step to reduce the number of values, obtaining as a result a step function, and (ii) transform this step function into a continuous PWL satisfying the FIFO property. The algorithm is efficient and rather simple to implement, but does not provide upper bounds regarding the error obtained from the estimation.

Recently, integer programming (IP) based methods for general continuous PWL functions have been proposed in [4,3]. Given a desired upper bound of the number of pieces, these methods are able to compute optimal estimations with respect the ℓ_∞, ℓ_1 or ℓ_2 norms. In general, the IP models are very flexible and can be adapted in order to pursue different objective functions, such as minimizing the number of pieces while respecting a maximum error for the estimating. As a counter part, they are more demanding in terms of computation times since the rely on IP models that are sensitive to the number of pieces to be estimated.

In this paper, we investigate the problem of estimating these continuous PWL travel time functions from real data travel time data. We benchmark two recently proposed Mixed Integer Programming based models for estimating general PWL functions and a well-known heuristic proposed within the context of travel-time estimations. In addition, we also contribute with a new dataset of instances created using real-world data as input.

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